ON THE CONVERSION OF TRANSLATIONAL INTO ROTATIONAL MOTION WITH THE SLIDER-ROCKER MECHANISM, REGARDING TRANSFER QUALITY

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ABSTRACT

The paper discusses the dimension synthesis procedure of the slider-rocker mechanism, that has the best possible transfer quality (transmission angle), according to the German directive VDI-2125. A new approach is proposed that considers acceptable transfer quality, thereby leaving more design freedom to the user. It appears to be useful that the coupler length is considered as the free design parameter. For this parameter the feasible range has been determined. Examples are presented showing that a great variety of mechanism solutions may exist, from which the designer can choose his favourite one, or can fullfil other demands. Diagrams have been developed to easily access the new method. It is recommended to update the existing VDI-directive with the new approach.

Keywords: Dimension Synthesis, Transmission Angle, Linear Transfer Function, VDI-2125

1. INTRODUCTION

Conversion of translational motion into an oscillating rotation can be done easily with a rack and a pinion, providing a linear kinematic transfer function. Occasionally however a designer prefers a different mechanism, for instance to avoid the backlash that is typical for a pair of gears. A link mechanism, like the planar slider-rocker mechanism, is the alternative mechanism with the simplest kinematic structure. The dimension synthesis of this mechanism, for optimum transmission angle, is the topic of the VDI-directive 2125 [1]. The aim of this paper is to discuss the theory of the existing synthesis procedure and to propose new ideas with which the directive can be improved.

The mechanism is depicted in figure 1, drawn in the three positions that play a role in the calculations. The synthesis problem can be described as follows:

Given are the input stroke s_H and a desired angular output stroke ψ_H, for which the four kinematic parameters (two bar lengths b and c, and the two co-ordinates of the fixed pivot point e and t) must be calculated. Further condition is that the transmission angle μ is "good". In the VDI-directive this is expressed, in its basic form, by applying the following conditions (relations of the parameters) extra to the design objective equation $s_H(\psi_H)$:

$\mu_1 = \mu_{min}$
$\mu_2 = \mu_{min}$
μ_{min} = maximum value

The transmission angle, in both end positions (numbered 1 and 2), must be equal to the minimum value, while this minimum value should be "as high as possible". The μ_{min} position is numbered as position 3 and is situated where the distance between the slider point A and the pivot point B_0 is minimal: B_0 lies on the normal to the slider path in that point. Angles μ_1 and μ_2 are defined as complement angles to be comparable directly with μ_{min}.

Comment on this approach: it is certainly a good idea to take the transmission angle as a measure for transfer quality. The driving of the slider will usually be done with a reciprocating cylinder. Because in the end-points the velocity is zero, static forces need to be considered in the first place (force transfer along the coupler).
The application of the four conditions, as previously described, leads to precisely one solution. One may ask whether or not the last condition, specifying max(μ_{min}), is too much limiting the application of the mechanism. Normally the user will be satisfied with a certain acceptable μ_{min} value, implying that one of the parameters can be chosen freely, within certain limits. The intention of this paper is then to investigate the theory of this idea and to present its application (chapter 2).
A side condition is that the transfer function $\psi(s)$ must not show backward motion: the output angle must be monotonic during the whole input stroke. In the VDI-directive it is proven that, due to this side condition, the basic problem has only a solution for a limited range of ψ_H up to 76.345°. When the condition $\mu_2 = \mu_{min}$ is dropped and replaced by $d\psi/ds = 0$ in the end-position 2, the range can be extended up to theoretically $\psi_H = 270°$. This extended synthesis problem will be treated in chapter 3.

2. BASIC SYNTHESIS PROBLEM

In the remaining chapters all parameters with dimension length will be understood relative to input stroke s_H ($s_H = 1$ will be assumed).

2.1 Theory according VDI-2125

Using the help quantities $f_1 = A_1B_0$ and $f_2 = A_2B_0$ according figure 1:

$$f_1^2 = e^2 + (1-t)^2 \tag{1}$$
$$f_2^2 = e^2 + t^2 \tag{2}$$

The transmission angle μ can be expressed in the three positions as:

$$\cos\mu_1 = -\frac{b^2 + c^2 - f_1^2}{2bc} \tag{3}$$
$$\cos\mu_2 = -\frac{b^2 + c^2 - f_2^2}{2bc} \tag{4}$$
$$\cos\mu_{min} = \frac{b^2 + c^2 - e^2}{2bc} \tag{5}$$

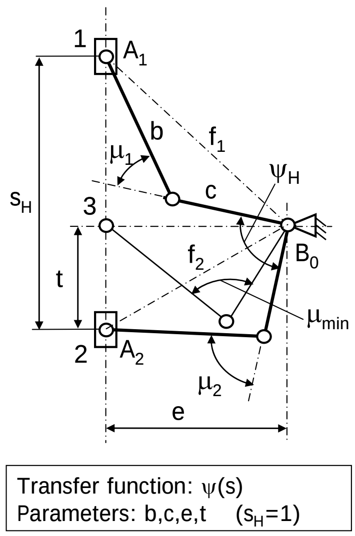

Fig. 1 Slider-rocker mechanism

The conditions $\mu_1 = \mu_{min}$ and $\mu_2 = \mu_{min}$ lead to the relations (synthesis equations) between the parameters:

$$(1-t)^2 = 2b^2 + 2c^2 - 2e^2 \tag{6}$$
$$t^2 = 2b^2 + 2c^2 - 2e^2 \tag{7}$$

Combining equations (6) and (7) yields $t = 0.5$: the fixed pivot point B_0 must lie on the perpendicular to slider path A_1A_2 through the midpoint. With this result it follows also that $f_1 = f_2$, and directly from figure 1:

$$e = \frac{t}{\tan(\psi_H/2)} = \frac{1}{2\tan(\psi_H/2)} \tag{8}$$

According to [1] the condition max(μ_{min}) has the solution:

$$b = c = \frac{1}{2}\sqrt{2e^2 + t^2} = \frac{1}{4\sin(\psi_H/2)}\sqrt{1+\cos^2(\psi_H/2)} \tag{9}$$

Example: for $\psi_H = 60°$ the solution is: $t = 0.5$, $e = 0.866$, $b = c = 0.661$ and $\mu_{min} = 81.8°$.
In [1] it has been derived that for $\psi_H > 76,345°$ the side condition for monotony will be violated, so this theory is applicable up to this ψ_H-value.

2.2 New theory

The example shows a transmission angle that may be felt as „too good“. What if a μ_{min}-value of 60° will be accepted? The condition of max(μ_{min}) can be skipped then and one of the parameters b or c can be chosen freely. Suppose coupler length b is considered as the new design variable. Using (7) the parameter c is determined:

$$c = \sqrt{\tfrac{1}{2}t^2 + e^2 - b^2} \tag{10}$$

Parameter b can be varied to determine the effect on the minimum transmission angle (5), see figure 2.
The region for b where $\mu_{min} > 60°$ appears to be:
$0.14 < b < 0.92$.
The side condition for monotony must however still be verified. This can be done by determining those b-values, as boundary values, satisfying the condition $d\psi/ds = 0$ in the end-position 2. In that case the coupler b is perpendicular to the slider path and this condition can be expressed by:

$$c^2 = t^2 + (e-b)^2 \tag{11}$$

Combining equations (7) and (11) yields the boundary values for b, expressed in the parameters e and t already known:

$$b_{1,2} = \tfrac{1}{2}\left(e \pm \sqrt{e^2 - t^2}\right) \tag{12}$$

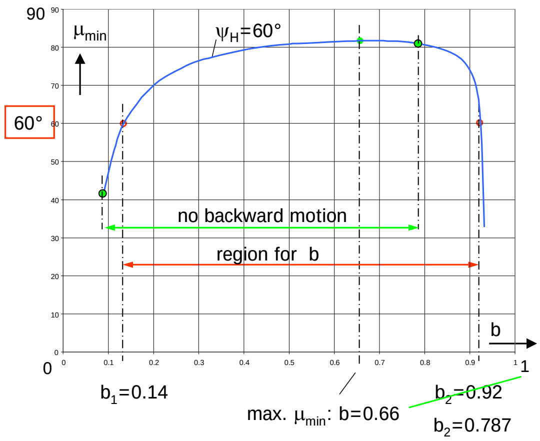

Fig.2 Example: $\mu_{min}(b)$ for ψ_H=60°

In the example $b_1 = 0.079$ and $b_2 = 0.787$. The usable region for b decreases thus to $0.14 < b < 0.787$.
To draw the complete μ_{min}-behaviour the whole range of b, for which μ_{min} is calculable, must be determined. The corresponding boundary values are defined by $\mu_{min} = 0$ (position 3). Here are two situations possible, see fig. 3: either c = b + e or c = b - e. Substitution into (7) leads to the calculable boundaries of parameter b:

$$b_{min} = \tfrac{1}{2}\left(\sqrt{e^2 + t^2} - e\right) \text{ and} \tag{13}$$

$$b_{max} = \tfrac{1}{2}\left(\sqrt{e^2 + t^2} + e\right), \tag{14}$$

in which t = 0.5 and and e is determined by (8) for a given value of ψ_H.
The diagram of figure 2 can be completed now for the whole range of ψ_H-values and b-ranges (see figure 4).

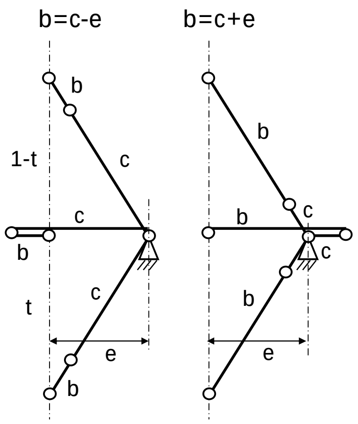

Fig.3 Configurations with μ_{min} = 0

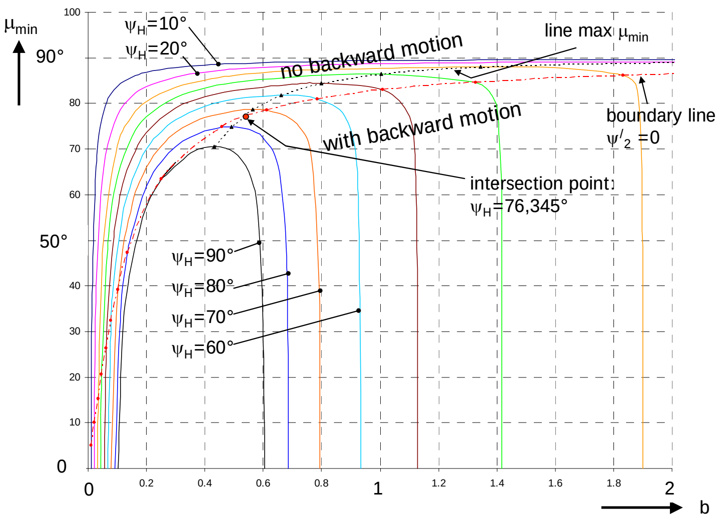

Fig.4 Diagram $\mu_{min}(b)$ for the basic synthesis problem

In this diagram the boundary line $\psi_2^{/} = 0$, defined by (12), has been drawn as well. Each ψ_H-line has two intersection points with this boundary line. Only the upper diagram part is feasible regarding the monotony condition. The ψ_H-values range from 0 to 90°. For $\psi_H = 90°$ the range of b reduces to a single point: b = 0.25 . The solutions of the existing VDI-directive are marked in the diagram by the line indicated with max(μ_{min}), which lies in the feasible part up to ψ_H = 76,345°.

The new approach shows that:

- The parameter b can be varied considerably while, especially for smaller values of the output angle ψ_H , the transfer quality is hardly decreased. This is mainly advantageous to obtain a smaller mechanism.
- The application range of ψ_H can be extended up to 90° for the basic synthesis problem.

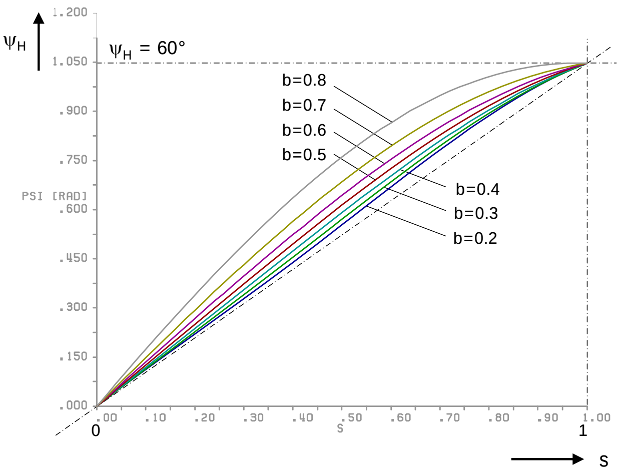

Fig. 5 Example: variety of transfer functions ψ(s) for ψ_H=60°

2.3 Verification

With an example the effect of variation of parameter b can be demonstrated. For the given output angle ψ_H = 60° solutions can be calculated for b-values ranging from 0.2 to 0.8. The diagram of figure 4 confirms that this range has feasible solutions (without backward motion) and the transmission angle is 70° or better. The transfer functions of these solutions are drawn in figure 5. It is obvious that the variety of the transfer functions will help a designer to make a choice for a proper b-value.

3. LARGER OUTPUT ANGLES (EXTENDED SYNTHESIS PROBLEM)

To obtain the largest possible output angle ψ_H, without backward motion, coupler b must be perpendicular to the slider path in the end-position 2. So when a large output angle is requested, it is a good idea to replace the condition $\mu_2 = \mu_{min}$ by the condition (11). For μ_2 the side condition $\mu_2 > \mu_{min}$ must still be maintained.

3.1 Theory according VDI-2125

The four equations to be solved concern: $\psi_H(s_H)$, equations (6) and (11), and the max(μ_{min}) condition. This set of equations appears to be solvable only numerically. It has been done in earlier work [1] and the results have been presented in a diagram, see figure 6. This diagram gives a nice overview of the transfer quality that can be reached maximally for a requested output angle. The parameter values to be obtained are however less accurate and additional correction (iteration by trial-and-error) is usually needed. In [1] these additional calculations concern the variation of the parameters e and/or t.

The author has extensively verified the results of [1], initially just to reconstruct the results of the diagram with accurate values, using a special computer program for optimization of mechanisms [2]. During this work it became obvious that the max(μ_{min})-condition always shows a week maximum. Another remarkable property of the diagram is that the value of parameter b is always close to 0.5. Eventually new insights were obtained: instead of finding the optimal b-value precisely, it is more valuable to determine the range of b with acceptable transfer quality.

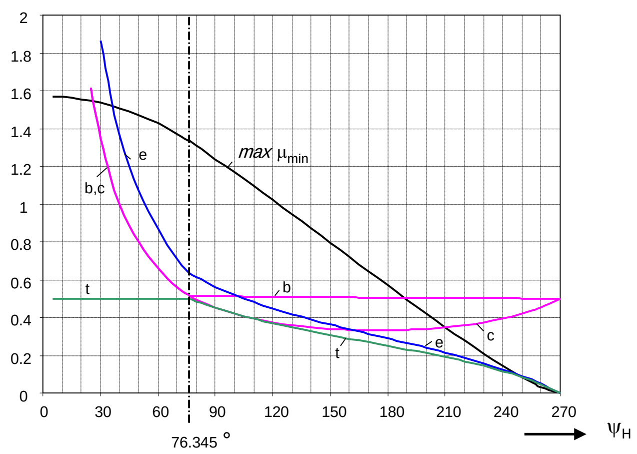

Fig. 6 Parameters and max (μ_{min}) acc. VDI-2125

3.2 New synthesis theory

Based on the new insights, the synthesis problem is to be modified. The max(μ_{min})-condition must be dropped and one of the parameters can be chosen freely, within certain limits. Parameter b was chosen to vary because it is already known that the optimum value is close to 0.5. The very first experiments using [2] were successful and it was chosen to develop the dedicated synthesis equations to solve the modified problem. Despite the equation system has been reduced now to three equations and three parameters, a closed solution could not be derived. It is however possible to derive an expression for one of the remaining parameters for which a root must be calculated numerically. The success of this method depends thus also on a proper start value of this parameter. With b given and e chosen as the „root“-parameter, parameter c follows directly from (11). The remaining parameter t can be calculated, which means expressed in b and e, using (6), after elimination of c:

$$t = \sqrt{2 + 4eb - 4b^2} - 1 \tag{15}$$

The equation of the transfer function to find the root for parameter e, reads, using also equations (1) and (2):

$$\psi_H = \arctan\frac{1-t}{e} + \arctan\frac{t}{e} + \arccos\frac{c^2 + f_2^2 - b^2}{2cf_2} - \arccos\frac{c^2 + f_1^2 - b^2}{2cf_1} \tag{16}$$

Using a general program for mathematical calculations [3] the results were identical to those obtained with [2]. To establish a diagram comparable to figure 4, the calculable range of b must be determined.
The value b_{min} is determined, see also figure 3, by $c = b + e$, where $\mu_{min} = 0$. Combined with equations (6) and (11) it is possible now, for this special situation, to express b, c and e in parameter t:

$$b = \tfrac{1}{2}\sqrt{1-2t} = b_{min} \tag{17}$$

$$c = \frac{(1-t)^2}{2\sqrt{1-2t}} \tag{18}$$

$$e = \frac{t^2}{2\sqrt{1-2t}} \tag{19}$$

Substitution in (16) yields a root equation for t: b_{min} can be determined numerically.
The value b_{max} is determined by $c = b - e$, but this situation applies only for $t = 0$ and $\psi_H \geq 180°$. For this situation a closed solution can be obtained, using also equations (6) and (11):

$$e = \frac{1}{\tan(\psi_H - 180°)} \tag{20}$$

$$b = \tfrac{1}{2}\left(\sqrt{1+e^2} + e\right) = b_{max} \tag{21}$$

$$c = \tfrac{1}{2}\left(\sqrt{1+e^2} - e\right) \tag{22}$$

Obviously $b_{max} = \infty$ when $\psi_H \leq 180°$.
The results as obtained above were used to create the diagram $\mu_{min}(b)$ for the extended synthesis problem, see figure 7.
Note that the max(μ_{min})-values occur indeed at about $b = 0.5$, but the function behaviour is far from symmetric to that point. The user may apply a much higher b-value without losing much transfer quality.
Note that the same boundary line as in figure 4 determines which part of the diagram is feasible. The infeasible part is here due to the violation of the side condition $\mu_2 > \mu_{min}$. For a user as intended by the VDI-directives it would be appropriate to combine the diagrams of figures 4 and 7, showing just the feasible regions of both synthesis problems.

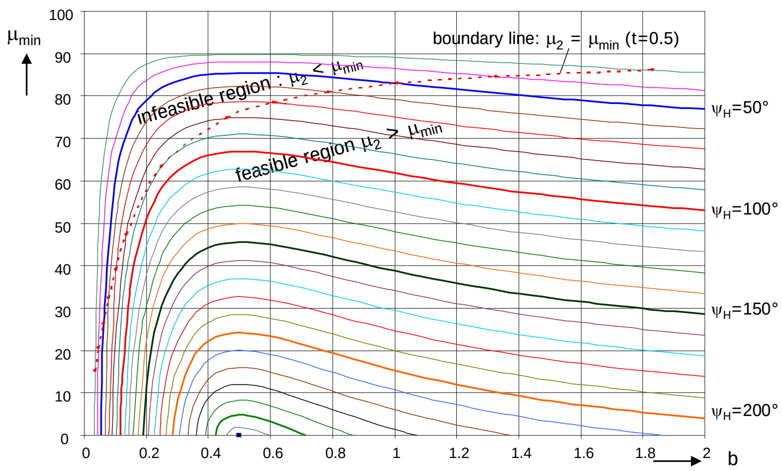

Fig. 7 Diagram $\mu_{min}(b)$ for extended synthesis problem

3.3 Feasible region of the pivot plane

The calculation procedure of the extended synthesis problem has been described in the previous paragraph. The user has sufficient information to perform the required calculations (for given values of ψ_H and b), either supported by a sophisticated calculation tool [3] or using a trial-and-error method to find the root of (16). But there is still one problem left: the start value of the parameter e of the root equation. During the production of the diagram of figure 7 this problem was frequently met and usually it was solved with the help of the previous solution. For a single calculation this information is not available, so it would be very helpful to have access to all calculated results done already. It is expected that this information, presented in a diagram in the pivot plane (e,t as co-ordinates) will help the user best.

To present a complete diagram it is also required to determine which part of the pivot plane is feasible. Some infeasible regions follow directly from the definition of the problem, like: $e < 0$ or $t < 0$. The region $t > 0.5$ violates the side-condition and must also be regarded as infeasible. Two further boundaries have been detected and they were investigated:

1) The boundary determined by $\mu_{min} = 0$. Actually this boundary is already specified by (19).

2) The condition that, for given values of e and t, the value of b and c are calculable. The synthesis equation (11) can be substituted in (6) to eliminate c, and this equation can be rewritten to express b as a function of e and t:

$$b_{1,2} = \tfrac{1}{2}\left(e \pm \sqrt{e^2 - t^2 - 2t + 1} \right) \tag{23}$$

The square root in (23) should not be negative. This yields a condition for feasibility:

$$t < \left(\sqrt{e^2 + 2} \right) - 1 \tag{24}$$

The two boundaries specified by the equations (19) and (24) are drawn in figure 8. They have a common contact point. To understand their effect two sample lines (e,t values for given ψ_H calculated for the full range of b-values) are drawn. To the left of the contact point, boundary $\mu_{min} = 0$ simply limits the feasible part of the pivot plane. The second solution of b is negative and can be neglected. To the right the situation is more complicated, due to the fact that (23) can have two solutions. Here the boundary according (24) limits the calculable region of the pivot plane. For $t > 0.5$ this part is to be excluded (side condition $\mu_2 < \mu_{min}$) but a small region with the double solution remains. The double solutions can also be detected in figure 7 (lower ψ_H-values only). Obviously the lower b-value is part of a solution with poor transfer quality. For a diagram serving a user this solution can be neglected. The full diagram showing all calculated results is presented in figure 9. It is easy now to find the pivot point B_0 approximately for a given value of angle ψ_H and a well-chosen b-value regarding proper transfer quality. To obtain the accurate synthesis result, the approximated value of e can be used in equations (15) and (16).

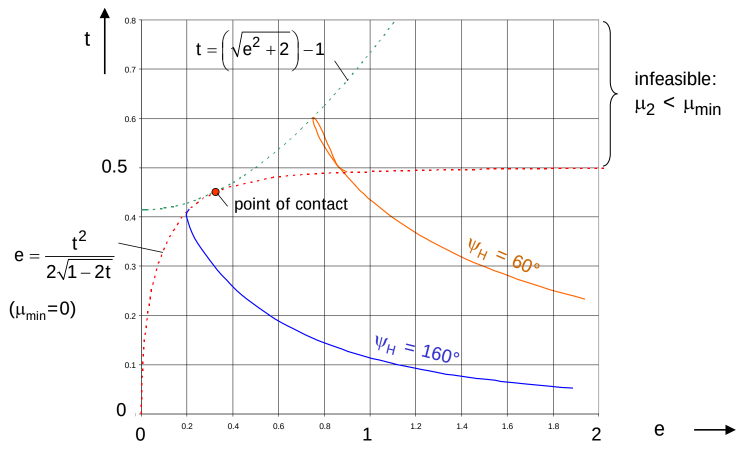

Fig. 8 Feasible region of pivot point (e,t –plane)

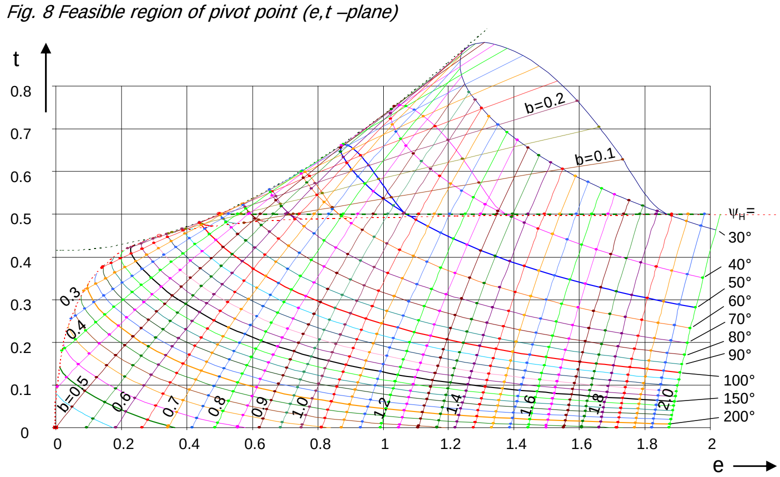

Fig. 9 Calculation help: diagram in pivot plane (e,t)

3.4 Example and verification

The synthesis equations (15) and (16) were placed in a spreadsheet. The example concerns the following input: $\psi_H = 160°$ and b = 0.7. From figure 7 it can be obtained that the transmission angle $\mu_{min} \approx 40°$. Figure 9 shows that the value of parameter e is approximately 0.52. By trial-and-error this value has been modified to 0.51216 to obtain the precise result, see table 1.

	1st trial	final trial
b	0.7	0.7
e	0.52	0.51216
t	0.22311	0.21410
c	0.28667	0.28482
ψ_H	155.5°	160°
μ_{min}	41.24°	39.24°

Table 1 Example of calculated results

The value b = 0.7 was an arbitrary choice. With other b-values the calculation can be repeated and the transfer functions can be compared. For this example the b-values are varied from 0.25 to 1.0, see figure 10. A designer will certainly be inspired to choose a favourable solution.

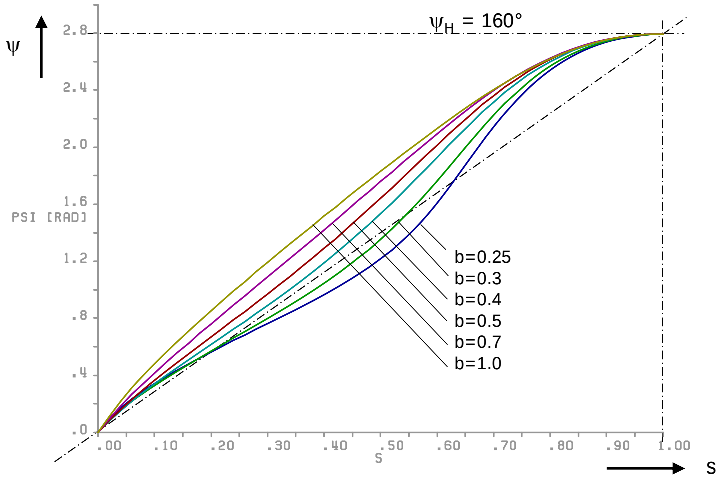

Fig. 10 Example: variety of transfer functions $\psi(s)$ for ψ_H=160°

4. LINEARITY AND OTHER USER DEMANDS

When a large output angle is demanded, the dead point in the end position may need discussion. Some applications do not allow a dead point, for instance when it concerns a controlled drive with feedback of the output motion. To obtain a good linearity of the transfer function, the (extended) synthesis procedure could still be used, but adapted as follows.

In a diagram as shown in fig. 10, a suitable transfer function can be selected that will be used partly. The last part of the input stroke, approaching the dead point, will not be used. The output angle ψ_H needs to be larger than required since only that part, corresponding to the reduced input stroke, will be used. The mechanism dimensions obtained with the synthesis procedure need to be enlarged proportionally to compensate for the input stroke reduction. This method is useful as long as the μ_{min}-position remains involved, because the condition $\mu_1 = \mu_{min}$ still makes sense then. This is the case when the input stroke reduction is less than parameter t.

A conclusion is that demanding linearity inevitably leads to other drawbacks, like reduced transfer quality and a larger mechanism. Nevertheless the proposed synthesis method helps to estimate these effects in the early design considerations.

The synthesis procedure leaves some freedom to put additional demands to the mechanism. Good linearity is just an example of such a demand. It is beyond the scope of this paper to discuss all options that can be imagined here to put additional demands. Frequently such demands can be formulated as motion conditions, leading to some kind of optimization problem. The computer program [2] is capable to solve such problems numerically. As solving an optimization problem needs a certain start solution, the synthesis results to be obtained with the method of this paper can be considered.

SUMMARY AND CONCLUSIONS

The synthesis procedure, as described in the VDI-directive 2125, unnecessary limits the application of the slider-rocker mechanism to convert the slider motion into a rotation, because of the focus on maximum transfer quality. It is better to consider an acceptable transfer quality, leaving some design freedom to the user.
The paper shows that parameter b (coupler length) is very well suited to demonstrate this design freedom. The range for b has been determined for which the transmission angle, as a measure for transfer quality, remains at least at a demanded value. The qualitative design decisions can be taken with the help of diagrams that have been developed for this purpose.
In case of an output angle larger than 90° (up to theoretically 270°) an alternative synthesis procedure can be applied, but this involves a dead point in the end position. The parameter calculations include finding the root of a non-linear equation. A diagram has been developed to show where the fixed pivot point needs to be chosen, dependent on the b-value to apply. This diagram also provides the estimated start value for the root calculation.
It is recommended to update the existing VDI directive with the new theory of this paper.

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