Motion Conversion with the Crank-Slider Mechanism regarding Transfer Quality (Part 1)

A.J. Klein Breteler

Univ. of Technology Delft, The Netherlands, e-mail: a.j.kleinbreteler@tudelft.nl

Abstract. The paper discusses the dimension synthesis procedure of the crank-slider mechanism, matching a given input angle and a desired output stroke, for the best possible transfer quality (highest minimum value of the transmission angle), according to the German guideline VDI-2126 (1989). A modified approach is proposed in which the transmission angle just needs to be acceptable. This leads to a much simpler synthesis procedure that covers the most relevant design criteria, like the minimum transmission angle, space occupation and occurrence of dead points. The link dimensions can be obtained from a diagram or can be calculated using simple formulas.

Key words: Dimension synthesis, Transmission angle, Space occupation, Dead point, VDI-2126.

1 Introduction

Conversion of oscillating rotation into translational motion can be done easily with a rack and a pinion, providing a linear kinematic transfer function. Occasionally however a designer prefers a different mechanism, for instance to avoid the backlash that is typical for a pair of gears. The planar crank-slider mechanism is the alternative with the simplest kinematic structure [1, 2]. Applying the crank angle as the input, the dimension synthesis of this mechanism, for the best possible transmission angle, is the topic of [1]. Because a part of the current theory is complicated, the aim of this paper is to discuss the theory of the synthesis procedure and to propose new ideas for improvement of the guideline.

The mechanism is depicted in Fig. 1, drawn in the three positions that play a role in the procedure. The synthesis problem can be described as follows.

Given are the angular input stroke φ_H and a desired output stroke s_H, for which the four kinematic dimensions (bar lengths r and b, and the co-ordinates of the fixed pivot e and t) must be determined. Further condition is that the transmission angle μ is "good" during the whole motion range. In the VDI-guideline this is expressed by applying the conditions $\mu_1 = \mu_2 = \mu_3$ (= μ_{min}), extra to the design objective equation that input angle φ_H corresponds to output stroke s_H.

In both end positions the transmission angle μ must be equal to the minimum value, while this minimum value should be as high as possible. The position 3, where the crank is perpendicular to the slider path, holds here the μ_{min}-value.

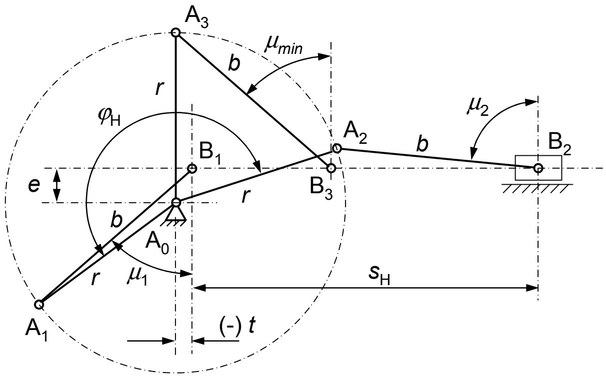

Fig. 1 Crank-slider mechanism in the three design positions

The ideal situation is that $\mu_{\min} = 90°$, but in general it is impossible to achieve this as the synthesis result. Which value of $\mu_{\min}$ is acceptable depends also on the use of the mechanism and on dynamic forces that are however unknown yet in the early design stage of the mechanism. That is why the acceptable value of $\mu_{\min}$ is usually chosen from experience with previous design cases. In practice frequently a minimum value of 60° or 45° will be adopted. Anyhow it is useful to know which $\mu_{\min}$-value can maximally be achieved.

A side condition is that the transfer function $s(\varphi)$ must not show backward motion: the output displacement must be monotonic during the whole motion of the input angle at interval φ_H. A dead point at one or both boundaries of the interval will be included in the theory. Dependent on the application, such a dead point can be in favour (e.g. as output lock) or needs to be avoided (e.g. in a controlled drive with output feedback). The precise behaviour of the transfer function on the interval will not be subject of discussion. In case that an approximated linear behaviour is wanted, some design freedom can be exploited to meet this requirement.

In case that $\varphi_H < 180°$, the problem can be solved easily using the so-called symmetric solution. This case has adequately been described in [1]. The crank positions 1 and 2 are then symmetric to the middle position 3 and the crank endpoints A_1 and A_2 have the same distance, but opposite, to the guiding line as A_3. Theoretically coupler length $b = \infty$ would provide the best transfer quality ($\mu_{\min} = 90°$), but this solution is not practical. The smallest possible value of b includes a dead point in the end-position. By choosing a proper finite value of b a compromise between transfer quality and space occupation must be accepted.

In case that $\varphi_H > 180°$ the symmetric solution is no longer possible due to monotony failure, but a non-symmetric solution demanding $\mu_1 = \mu_3$ ($= \mu_{\min}$), while $\mu_2 > \mu_{\min}$, can still be obtained. It is the intention of this paper to describe the design problem of this case and to present the solution options for a designer. This will be done by means of a graphical approach (chapter 2), after which the required parameter calculations will be specified (chapter 3). To support the user a diagram will be proposed that overviews the design options and that also provides the parameter values of design cases with and without dead points (chapter 4).

2 The Non-symmetric Solution, Graphical Approach

This chapter deals with a non-symmetric solution and includes the case of larger input angles (180° < φ_H < 270°). We split up the input angle φ_H into two angles α and β, see also Fig. 2:

$$\varphi_H = \alpha + \beta, \text{ with } \alpha > \beta \quad (1)$$

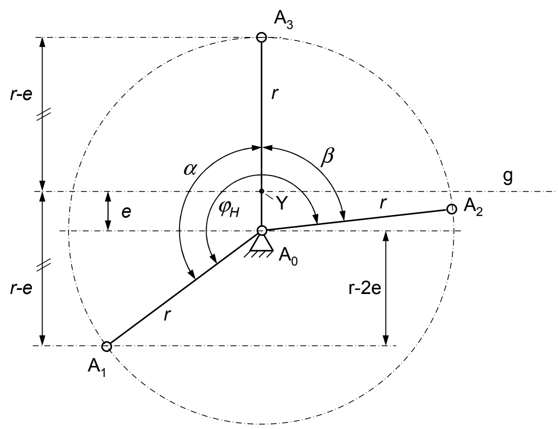

Fig. 2 Non-symmetric solution: Choose α and β

We assume that $\varphi_H/2 < \alpha < 180°$ and that $0° < \beta < 90°$. Instead of the transmission angle μ we consider the distance d of point A to the guiding line g in the three positions involved. The maximum value of d should be as low as possible: $d_1 = d_3$ (= d_{max}), while $d_2 < d_{max}$.

For any choice of α the configuration of Fig. 2 can be drawn. Start for instance by drawing the crank in the intermediate position 3 (the crank length $A_0A_3 = r$ is arbitrary and is considered as the drawing unit) and apply the angles α and β according Eq. (1) to find the points A_1 and A_2. Then the guiding line g can be determined: perpendicular to A_0A_3 and at equal distance to A_1 and A_3. Now the dimension $e = A_0Y$ is known and the distance $d_1 = d_3 = r - e$. Alternatively the value of e/r can be chosen from which point A_1 and thus α can be constructed.

Next step is to determine the range of length b, see Fig. 3. The largest possible value $b_{max} = A_1X$, otherwise the crank will surpass the dead point at position 1. The smallest possible value is at least $b_{min} = r - e$, for which $\mu_{min} = 0$.

Applying $r + b_{\min} = 2r - e$ and $r + b_{\max}$ as radii of concentric circles around A_0, we find the intersection points with the guiding line g named S_1 and S_2. When, for the chosen α, the angle β is such that the elongation of A_0A_2 intersects g between S_1 and S_2, the minimum value of b must be increased to A_2B_2 as drawn (solution with dead point at end position 2). Smaller values of coupler length b would cause backward motion of the slider. In case that angle β is smaller (the elongation of A_0A_2 intersects g before S_1) a valid solution may still exist, yet without a dead point in position 2. With a larger value of angle β, such that A_0A_2 intersects g beyond S_2, no valid range of b remains.

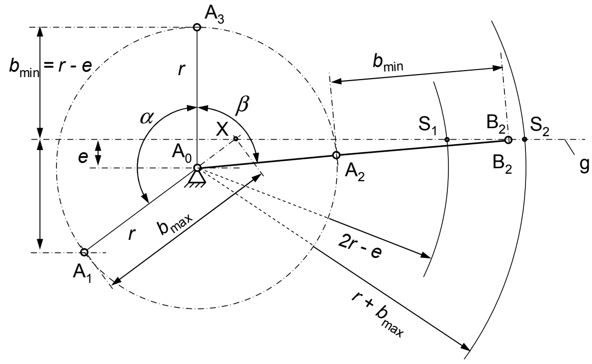

Fig. 3 Range of coupler length b

It is obvious that the initial choice of α has great effect on the range to choose coupler length b. In case of an invalid result the procedure must be repeated with a better (larger) value of α. In general it can be stated that a larger b-value will provide a higher transfer quality.

The third step concerns the drawing of the mechanism, with the dimensions of e and b obtained during the previous two steps, in the three positions as depicted in Fig. 1. This determines also the start position of the slider (point B_1, dimension t) and the end-position (point B_2). The output stroke s_H/r is thus known, that means it can be measured on the drawing relative to crank length r. Scaling the drawing, such that s_H corresponds to the demanded value (applying the ratio of s_H and s_H/r), provides crank length r and thus the other dimensions.

3 The Non-symmetric Solution, Parameter Calculation

In the previous chapter it appeared that a simple synthesis procedure can be followed when all dimensions are initially taken relative to crank length r. This length can be considered as the scaling value to achieve the desired output stroke s_H. We introduce thus the parameters e/r, b/r and t/r for the dimensional synthesis.

At least the equation $d_1 = d_3$ must be satisfied, yielding the following relation between angle α and parameter *e/r*:

$$\cos\alpha = 2\cdot(e/r)-1 \tag{2}$$

Applying Eq. (2) two parameters still can be chosen freely. Each required dead point reduces the number of free parameters by one. Several situations on dead points can be distinguished and they will be described below. Focus will be laid on the determination of the two parameters *e/r* and *b/r*. The calculation of the third parameter *t/r* and the relative output stroke s_H/r is straightforward and parallel to the graphical description in the previous chapter.

In case that a dead point both at start and at end is required, the parameter values can be calculated as follows. The dead point at the start position requires that

$$\cos\alpha = \frac{e-r}{b} \tag{3}$$

Combination of the Eq. (2) and Eq. (3) yields a relation between *e/r* and *b/r*:

$$b/r = \frac{r-e}{r-2\cdot e} = \frac{1-e/r}{1-2\cdot e/r} \tag{4}$$

The dead point at the end requires that (see Fig. 3, position 2)

$$\cos\beta = \frac{e}{r+b} = \frac{e/r}{1+b/r} \tag{5}$$

Using Eq. (1) and substituting Eqs.(2), (4) and (5) an equation in the unknown parameter *e/r* results:

$$\varphi_H = \arccos\left(2\cdot e/r-1\right)+\arccos\left(\frac{(e/r)\cdot(1-2\cdot e/r)}{2-3\cdot e/r}\right) \tag{6}$$

This equation needs to be solved numerically (root calculation, e.g. regula falsi). Extensive trials have learned that a start value $e/r = 0$ will always do and that only one solution will be found. The result of Eq. (6) can be applied in Eq. (4) to find the value of *b/r*. It is the solution with the highest possible value of both *e/r* and *b/r* and also with the highest transfer quality, see later.

If only a dead point at the end (position 2) is required, one parameter can be chosen freely. In the previous chapter it became already clear that the value of *e/r* can be chosen within a certain small range. The highest possible value of *e/r* has been found by solving Eq. (12). The lowest possible value of *e/r* is determined by coincidence of the points B_2 and S_1, see Fig. 3:

$$b/r = 1 - e/r \tag{7}$$

Using again Eq. (1) and substituting Eqs. (2), (5) and (7) the following equation in the unknown lowest possible value of *e/r* can be obtained:

$$\varphi_{\mathrm{H}} = \arccos\left(2 \cdot e/r - 1\right) + \arccos\left(\frac{1 - e/r}{2 - e/r}\right) \tag{8}$$

This equation has to be solved numerically. Extensive trials have learned that a start value $e/r = 0$ will always do and that only one root will be found.

When a user has chosen a certain value of *e/r*, within the range as specified above, the corresponding value of *b/r* can be calculated, using Eq. (2), see Fig. 3:

$$b/r = \frac{e/r}{\cos(\varphi_H - \alpha)} - 1 \tag{9}$$

Now the range of both parameters *e/r* and *b/r* has been determined for a given value of crank angle φ_{H}. Note that the better solutions are close to the highest possible value of *e/r*. Approximate maximum and minimum values of *e/r*, like available through a diagram (see later) are usually sufficient for practical use. It is not required then to solve Eq. (6) or Eq. (8). To obtain an accurate solution, the application of Eqs. (2) and (9) are the only calculations required!

In case that a dead point only in the start position is required, there is one parameter free. Eq. (4) specifies a direct relation between the parameters *e/r* and *b/r*. One of them can be chosen and the other can be calculated, no matter the value of φ_{H}. Regarding the range of *e/r* it can be stated that any value lower than the result of Eq. (6) provides a valid solution. Eq. (8) does not play a role here.

In case that no dead points are required, the two parameters *e/r* and *b/r* can be chosen freely within certain limits. The area of free choice appears in the diagram of Fig. 5, below the φ_{M}-line indicating the solution with a dead point at the end.

4 Diagram

Based on the theory as described above, a diagram has been developed, the two parameters *e/r* and *b/r* along the axes, that provides an overview of possible solutions. The symmetric solutions are included in this diagram as well. Area I contains the symmetric solutions, area II the non-symmetric ones.

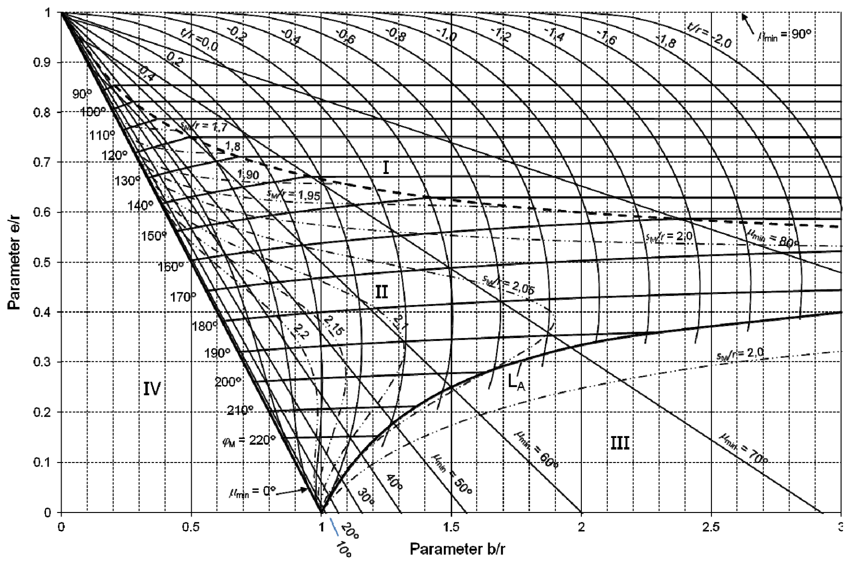

Fig. 5 Diagram

The $\mu_{\min}$-lines are straight lines with direction $\cos\mu_{\min}$, intersecting at (0,1). This can be derived immediately from Fig. 1, mechanism in position 3:

$$e/r = 1-(b/r)\cdot\cos\mu_{\min} \tag{10}$$

The boundary line L_A: Eq. (4) provides the relation between the parameters *e/r* and *b/r*. The equation can be rewritten as Eq. (11), in which the value of *b/r* is between one (*e/r* = 0) and infinity (*e/r* = 0.5).

$$e/r = \frac{b/r-1}{2\cdot b/r-1} \quad ; \quad 0 \le e/r \le 0.5 \tag{11}$$

The boundary line $L_{I\text{-}II}$ between symmetrical and unsymmetrical solutions (dashed): In accordance with [1], the symmetrical solution, Eq. (12) can be derived. Note that $\mu_{\min} = \varphi_H/2$ and that for *b/r* in infinity *e/r* = 0.5.

$$e/r = \frac{1 + b/r}{1 + 2 \cdot b/r} \quad ; \quad 0.5 \le e/r \le 1 \tag{12}$$

The φ_M-lines: For region II these lines have been described in chapter 3 (solution with a dead point at the end). Note that such solutions also exist for $\varphi_M < 180°$! In region I of symmetric solutions these lines continue horizontally, but they identify here ordinary solutions without dead point.

The *t/r*-lines: Clearly parameter *t/r* is completely determined by a point in the diagram: the values of *e/r* and *b/r* can be applied in the start postion 1 of the mechanism, using also Eq. (2). A constant value *t/r* is thus a line in the diagram.

The s_M/r-lines in region II (dash-dotted): They deal with the special situation that a dead point at the end will occur. For a given point in the diagram (all parameters known!) the s_M/r-value is also known. A constant value of s_M/r is thus a line in the diagram, showing the output stroke relative to the mechanism size.

5 Conclusions

The synthesis procedure to obtain a crank-slider mechanism as described in [1] has been investigated. It deals with monotonic transfer of motion from the crank (input) to the slider (output) such that the transmission angle is as good as possible. The current procedure for larger input angles requires that a user specifies a demanded transmission angle exactly. This leads to a complicated equation system that needs (nested) iterative calculations. The category of solutions has been limited to those mechanisms that have a dead point at the end of the motion interval. The existence of a dead point at the start of the interval has completely been neglected.

The paper shows that a much simpler procedure can be developed when the transfer quality does not need to be specified exactly, but is used to guide the possible design options for the user like choosing or avoiding a dead point at the start or at the end of the motion interval, the transfer quality and the size of the mechanism. To calculate the mechanism dimensions a set of simple equations will do. A diagram has been developed that overviews the design options.

The newly proposed synthesis procedure should be preferred and the current guideline VDI-2126 can be improved with the results of this paper.

6 References

1. NN: Planar mechanisms, Transfer of a rocker motion into a slider motion with regard to optimum transmission angle. VDI-guideline 2126, May 1989 (VDI-Richtlinie 2126, German).
2. NN: Planar Mechanisms, Transfer of a slider motion into a rocker motion with regard to optimum transmission angle. VDI-guideline 2125, May 2014 (VDI-Richtlinie 2125, German).